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LETTER TO THE EDITOR

Intra-sub-band and inter-sub-band collective excitations in a quasi-(0 + 1)-dimensional superlattice

Danhong Huang† and Shixun Zhou‡

† China Centre of Advanced Science and Technology (World Laboratory), PO Box 8730, Beijing, People's Republic of China and Department of Physics, Fudan University, Shanghai, People's Republic of China

‡ Department of Physics, Fudan University, Shanghai, People's Republic of China

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Abstract. We have developed a quantum theory of tunnelling plasmons in a quasi-(0 + 1)-dimensional superlattice (also called a lateral multiwire quantum dot superlattice). The analytical expressions of dispersion for both intra-sub-band and inter-sub-band tunnelling plasmons are derived using a tight-binding approximation. The connection between a quasi-(0 + 1)-dimensional superlattice and an anisotropic array of quantum dots is discussed.

Recently, the successful fabrication of a dot superlattice has been reported by Reed *et al* [1]. Also, the fabrication of quasi-(1 + 1)-dimensional superlattices, using a grating-like gate on a silicon inversional layer, was realised by Warren *et al* [2] and Hansen *et al* [3], respectively. Huang *et al* [4] have proposed a quantum theory of tunnelling plasmons in a quantum dot GaAs–$Al_{1-x}Ga_xAs$ superlattice. Here we would like to extend our previous results [4] to a more general case, in which the quasi-(0 + 1)-dimensional superlattice is within the xy plane, and the direction of the wire is along the x direction, as shown in figure 1. The lateral sizes of the dot superlattice are L_y and L_z. The period of the dot superlattice, along the x direction, is d, and the distance between the adjacent wires, along the y direction, is a. The electron motions along the x and y directions are tunnelling and electrically insulated, respectively. The single-particle states and the energy levels are

$$|k_x, m, j\rangle = \Phi_j(y - ma)\eta(z)\left(N^{-1/2}\sum_L \exp(\mathrm{i}k_xLd)\xi_\mathrm{p}(x - Ld)\right) \tag{1}$$

$$E(k_x, j) = W(1 - \cos k_xd)/2 + E_j \tag{2}$$

with a variational wave function [5] in the z direction

$$\eta(z) = (2L_z^3)^{-1/2} z \exp(-z/2L_z) \tag{3}$$

where $j = 0, 1, 2, \ldots$ is the sub-band index. We have assumed that the system will be always in its lowest state with respect to confinement in the z and x directions, and no excitations in these two directions will be considered due to the general experimental conditions $d, L_z \ll L_y$ [2, 3]. Following the SCF scheme of Ehrenreich and Cohen [6] and

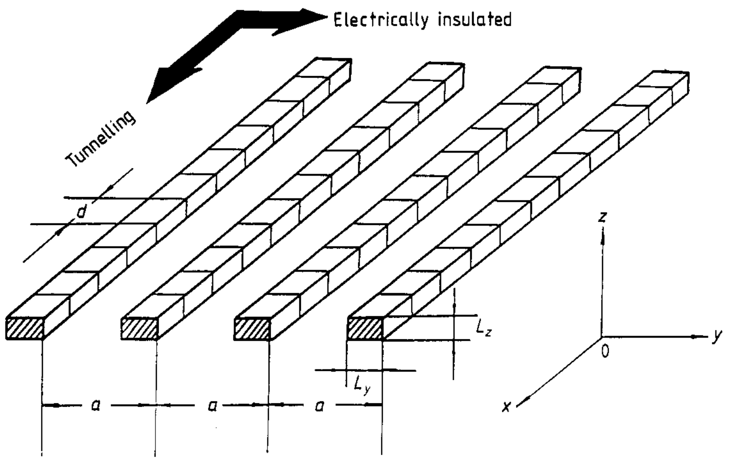

Figure 1. Quasi-(0 + 1)-dimensional superlattice scheme, where d and a are the periods of the superlattice along the x and y directions, respectively. L_y and L_z are the lateral sizes of the wires. The system is electrically insulated along the y direction, and allows electron tunnelling along the x direction.

using the Bloch condition [7] in the y direction, we finally get the dispersion relation after a lengthy manipulation similar to that in [4], that is,

$$\det\left|\delta_{jj'} - (2\pi e^2/\varepsilon_s)\sum_{j'}\chi_{j'0}(q_x,\omega)\sum_{n,L} I\{[(q_x+LG_1)^2+(q_y+nG_2)^2]^{1/2}\}[(q_x+LG_1)^2 + (q_y+nG_2)^2]^{-1/2}|A(q_x+LG_1)|^2 B^*_{j0}(q_y+nG_2)B_{j'0}(q_y+nG_2)\right| = 0 \tag{4}$$

where

$$I(q) = \int \mathrm{d}z\,\mathrm{d}z'\exp(-q|z-z'|)\,|\eta(z)|^2\,|\eta(z')|^2 = [8+9qL_z+3(qL_z)^3]/[8(1+qL_z)^3] \tag{5}$$

$$A(q) = \exp(-q^2d^2/8). \tag{6}$$

$G_1 = 2\pi/d$ and $G_2 = 2\pi/a$ are the reciprocal-lattice vectors along the x and y directions, respectively. $\chi_{j0}(q_x,\omega)$ is the polarisability given by

$$\chi_{j0}(q_x,\omega) = [2(n_0-n_j)\Omega_{j0}/\hbar d(\omega^2-\Omega^2_{j0})] + [4W\sin^2(q_xd/2)(\omega^2+\Omega^2_{j0})\sin(k_Fd)]/[\pi\hbar^2 d(\omega^2-\Omega^2_{j0})^2] \tag{7}$$

where n_j (n_0) is the linear electron density per wire for electrons in the jth (ground) sub-band, and Ω_{j0} is the energy difference between the jth and the ground sub-bands. It has been proved [8] that in the strong-confinement limit the confining potential has a parabolic shape. For less strong confinement the bottom of the potential resembles a square well. We consider the simplest case where there are only two sub-bands. This is a simple approximation introduced in [4], which gives a basic and understandable physical insight into the system. Furthermore, using a parabolic shape potential model we get the intra-sub-band and the dominant inter-sub-band plasmons

$$\omega_0^2 = [4W\sin^2(q_x d/2)\sin(k_F d)D_{00}(q_x, q_y)]/(\pi\hbar^2 d) \tag{8}$$

$$\omega_{10}^2 = [\Omega_{10}^2 + \Omega_+^2 D_{10}(q_x, q_y)/2] \pm [\Omega_+^2 D_{10}(q_x, q_y)/2]\{1 + 4\Omega_{10}^2(\Omega_+^2 - \Omega_-^2)/[\Omega_+^4 D_{10}(q_x, q_y)]\}^{1/2} \tag{9}$$

where

$$\Omega_\pm^2 = 2n_0\Omega_{10}/(\hbar d) \pm [4W\sin^2(q_x d/2)\sin(k_F d)]/(\pi\hbar^2 d) \tag{10}$$

$$D_{10}(q_x, q_y) = (2\pi e^2/\varepsilon_s)\sum_{n,L} I\{[(q_x + LG_1)^2 + (q_y + nG_2)^2]^{1/2}\} \times [(q_x + LG_1)^2 + (q_y + nG_2)^2]^{-1/2}|A(q_x + LG_1)|^2\,|B_{10}(q_y + nG_2)|^2. \tag{11}$$

$I(q)$ and $|A(q)|^2$ have been given above by equations (5) and (6), and $|B_{10}(q_y + nG_2)|^2$ is given by

$$|B_{00}(q_y)|^2 = \exp[-\hbar q_y^2/(2m\Omega_{10})] \tag{12}$$

$$|B_{10}(q_y)|^2 = [\hbar q_y^2/(2m\Omega_{10})]\exp[-\hbar q_y^2/(2m\Omega_{10})]. \tag{13}$$

In principle we can include all the sub-bands in the numerical calculation of (4). We would like to point out that there still exist other inter-sub-band plasmons associated with the $\Delta N = 1$ transitions in the system if we take more than two sub-bands into account, but their energies lie close to the sub-band-energy separation E_{10} and can be easily decayed, other than for the inter-sub-band plasmon given above by (9), which has a large depolarisation shift from the sub-band energy separation, and corresponds to the largest electric dipole moment. Thus it can couple strongly to the experimental probe.

It should be noted that when L_z tends to zero, the whole system will approach an anisotropic array of quantum dots. Thus we can get the dispersion from (8) and (9) by setting $I(q) = 1$. It is evident that the quasi-(0 + 1)-dimensional superlattice exhibits crossover in structure from one-dimensional to two-dimensional systems, as discussed in [4]. The situation strongly depends on the lateral sizes L_y, L_z and the distance between the adjacent wires. Also, the superlattice exhibits crossover in structure from quasi-(0 + 1)-dimensional to quasi-(1 + 1)-dimensional systems [9] when the period of the dot superlattice tends to zero. Moreover, from (8) we can see an important feature of the intra-sub-band plasmon near the Γ-point, in which the plasmon is completely 'softened' as $q_x \to 0$. It is attributed to the dramatic enhancement of the complete Coulomb screening due to the in-phase motion of electrons along the x direction. This is analogous to the longitudinal in-phase oscillation of a one-dimensional atomic chain. The spectrum also presents a periodic feature, which implies that the density of states of the plasmon will exhibit periodicity in two-dimensional $\boldsymbol{k}$-space.

When $W = 0$, we will get the usual inter-sub-band plasmon mode in (9), but when $\Omega_{10} = 0$, on the other hand, we will get the so-called tunnelling plasmon mode. In figure 2, both the tunnelling intra-sub-band plasmon mode (upper) and the coupled tunnelling inter-sub-band plasmon mode (lower) are presented. The oscillating and the splitting features in the lower part are attributed to the coupling between the inter-sub-band and tunnelling plasmon modes. Also, the depolarisation shift of the coupled tunnelling inter-sub-band plasmon mode is evident.

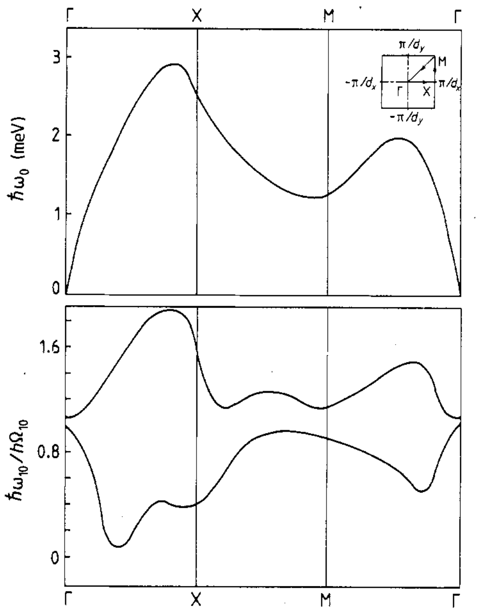

Figure 2. The spectra of the tunnelling intra-sub-band plasmon mode (upper) and coupled tunnelling inter-sub-band plasmon mode (lower). The definitions of Γ, M and X are shown in the inset of the upper part. The parameters used in the calculation are as follows: $W = 2.5$ meV, $\hbar\Omega_{10} = 15$ meV, $\varepsilon_s = 6.5$, $n_0 = 1.58 \times 10^5$ cm^{-1}, $m = 0.067m_e$, $a = 500$ Å, $L_z = 30$ Å, $d = 50$ Å.

Although there has not yet been any report of the successful fabrication of this structure, we believe this fabrication to present no fundamental difficulty in principle, and the dramatic feature of the tunnelling plasmon predicted here in the quasi-(0 + 1)-dimensional superlattice should be experimentally observable.

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